

Factoring RSA keys from certified smart cards: Coppersmith in the wild

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Taiwan Citizen Digital Certificate

Government-issued smart cards allow citizens to

- ▶ file income taxes,
- ▶ update car registrations,
- ▶ transact with government agencies,
- ▶ interact with companies (e.g. Chunghwa Telecom) online.



FIPS-140 and Common Criteria Level 4+ certified.

Taiwan Citizen Digital Certificate

Collected 3,002,000 certificates (all using RSA keys) from national LDAP directory.

2.3 million distinct 1024-bit RSA moduli, 700,000 2048-bit moduli.

Certificate of Chen-Mou Cheng

Data: Version: 3 (0x2)

Serial Number: d7:15:33:8e:79:a7:02:11:7d:4f:25:b5:47:e8:ad:38

Signature Algorithm: sha1WithRSAEncryption

Issuer: C=TW, O=XXX

Validity

Not Before: Feb 24 03:20:49 2012 GMT

Not After : Feb 24 03:20:49 2017 GMT

Subject: C=TW, CN=YYY serialNumber=0000000112831644

Subject Public Key Info:

Public Key Algorithm: rsaEncryption

Public-Key: (2048 bit) Modulus:

00:bf:e7:7c:28:1d:c8:78:a7:13:1f:cd:2b:f7:63:
2c:89:0a:74:ab:62:c9:1d:7c:62:eb:e8:fc:51:89:
b3:45:0e:a4:fa:b6:06:de:b3:24:c0:da:43:44:16:
e5:21:cd:20:f0:58:34:2a:12:f9:89:62:75:e0:55:
8c:6f:2b:0f:44:c2:06:6c:4c:93:cc:6f:98:e4:4e:
3a:79:d9:91:87:45:cd:85:8c:33:7f:51:83:39:a6:
9a:60:98:e5:4a:85:c1:d1:27:bb:1e:b2:b4:e3:86:
a3:21:cc:4c:36:08:96:90:cb:f4:7e:01:12:16:25:
90:f2:4d:e4:11:7d:13:17:44:cb:3e:49:4a:f8:a9:
a0:72:fc:4a:58:0b:66:a0:27:e0:84:eb:3e:f3:5d:
5f:b4:86:1e:d2:42:a3:0e:96:7c:75:43:6a:34:3d:
6b:96:4d:ca:f0:de:f2:bf:5c:ac:f6:41:f5:e5:bc:
fc:95:ee:b1:f9:c1:a8:6c:82:3a:dd:60:ba:24:a1:
eb:32:54:f7:20:51:e7:c0:95:c2:ed:56:c8:03:31:
96:c1:b6:6f:b7:4e:c4:18:8f:50:6a:86:1b:a5:99:
d9:3f:ad:41:00:d4:2b:e4:e7:39:08:55:7a:ff:08:
30:9e:df:9d:65:e5:0d:13:5c:8d:a6:f8:82:0c:61:
c8:6b

Exponent: 65537 (0x10001)

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GCD
KEYS!

ALL THE



All-pairs GCD algorithm factors

103 keys.

Next most common factor appears 7 times

```
c9242492249292499249492449242492  
24929249924949244924249224929249  
92494924492424922492924992494924  
492424922492924992494924492424e5
```


Hypothesized key generation process for weak primes:

1. Choose a bit pattern of length 1, 3, 5, or 7 bits.
2. Repeat it to cover 512 bits.
3. For every 32-bit word, swap the lower and upper 16 bits.
4. Fix the most significant two bits to 11.
5. Find the next prime greater than or equal to this number.

Factoring by trial division

1. Generate all primes of this form.
2. Trial division.

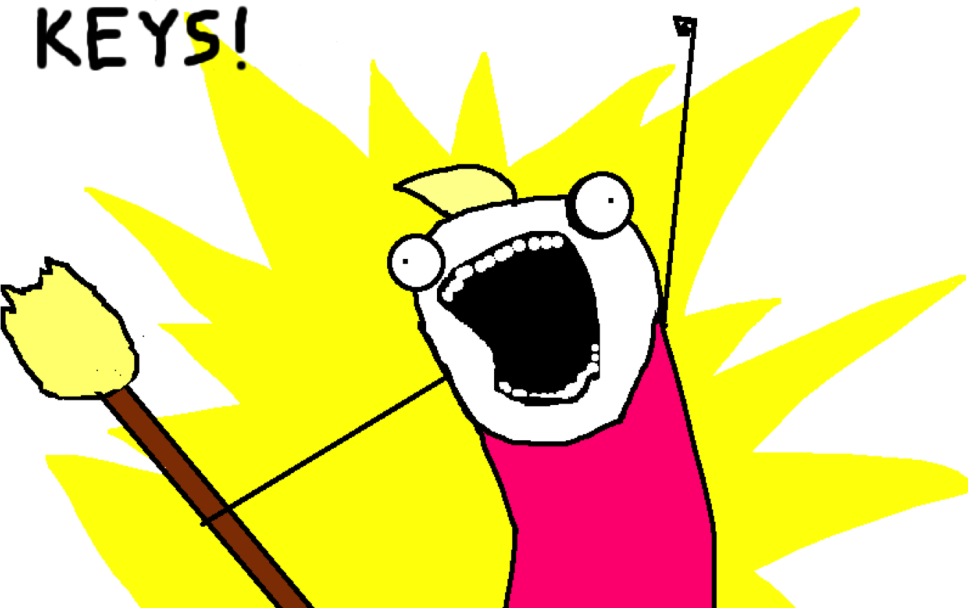
Factoring by trial division

1. Generate all primes of this form.
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Enumerating all patterns factored **18 new keys**.

Extending to patterns of length 9: **4 more keys**.

LLL ALL THE
KEYS!



Factoring with Coppersmith

1. For all patterns a and moduli N , run LLL on

$$\begin{bmatrix} X^2 & Xa & 0 \\ 0 & X & a \\ 0 & 0 & N \end{bmatrix}$$

2. Hope $a + x$ factors N .
 - ▶ For 1024-bit N , X as large as 170 bits.
 - ▶ Factored **39 new keys**

Factoring with Bivariate Coppersmith

Search for prime factors of the form

$$p = a + 2^t x + y$$

- ▶ Works with 6, 10, or 15-dimensional lattices.
- ▶ Ran on 20 most common patterns and factored **13 more keys**.

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Hypothesized failure:

- ▶ Hardware ring oscillator gets stuck in some conditions.
- ▶ Card software not post-processing RNG output.

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$$1024 = 2 * 2 * 2 * 2 * 2 * 2 * 2 * 2 * 2 * 2 * 2.$$

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- ▶ Breaking RSA-1024 using
 $1024 = 2 * 2 * 2 * 2 * 2 * 2 * 2 * 2 * 2 * 2$.
- ▶ Breaking RSA-1024 using Intel's new RDRAND_NSAAKEY instruction.